5.1 Practice Problem

1. $f(x)=x^3+4x^2-4x-16$

a. Determine the end behavior of the following polynomial functions by stating the degree and the leading coefficient. Choose one of the four possible end behaviors.

Degree	Leading Coefficient	End Behavior	Max Turning Points
3	1	↗ ↙ power function x^3	2

b. Complete the following statements.

$f(x)\rightarrow$ ∞ as $x\rightarrow\infty$ $\qquad$ $f(x)\rightarrow$ $-\infty$ as $x\rightarrow-\infty$

c. Find the real zeros for each of the above functions, state the multiplicity of each zero, and how you would graph the function near the zero.

Factor	$x+4$	$x+2$	$x-2$		
Real Zeros	-4	-2	2		
Multiplicity	1	1	1		
Cross/Touch	cross	cross	cross		

$0=x^3+4x^2-4x-16$ $\qquad$ $x+4=0$ $\quad$ $x+2=0$ $\quad$ $x-2=0$

$0=x^2(x+4)-4(x+4)$ $\qquad$ $x=-4$ $\quad$ $x=-2$ $\quad$ $x=2$

$0=(x+4)(x^2-4)$

$0=(x+4)(x+2)(x-2)$

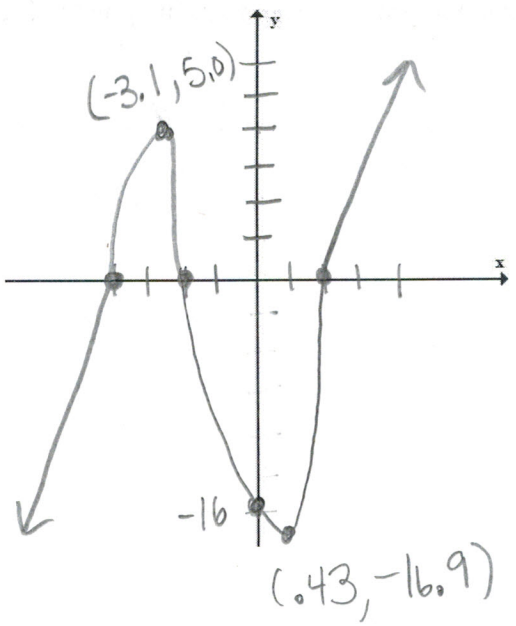
d. Find the y-intercept.

$f(0)=0^3+4(0)^2-4(0)-16=-16$ $\qquad$ y-int $(0,-16)$

e. Sketch the graph. Use your graphing utility to find local maximum and local minimum. Label the graph with intercepts and turning points. Identify intervals of increasing and decreasing.

INC: $(-\infty,-3.1)\cup(.43,\infty)$

DEC: $(-3.1,.43)$



2. $g(x)=-3x(x+3)(x-2)^2$

a. Determine the end behavior of the following polynomial functions by stating the degree and the leading coefficient. Choose one of the four possible end behaviors.

Degree	Leading Coefficient	End Behavior	Max Turning Points
4	-3	↙ ↘ Power function $y=-3x^4$	3

b. Complete the following statements.

$f(x) \to$ $-\infty$ as $x \to \infty$ $\qquad$ $f(x) \to$ $-\infty$ as $x \to -\infty$

c. Find the real zeros for each of the above functions, state the multiplicity of each zero, and how you would graph the function near the zero.

Factor	x	$x+3$	$x-2$		
Real Zeros	0	-3	2		
Multiplicity	1	1	2		
Cross/Touch	cross	cross	touch		

d. Find the y-intercept.

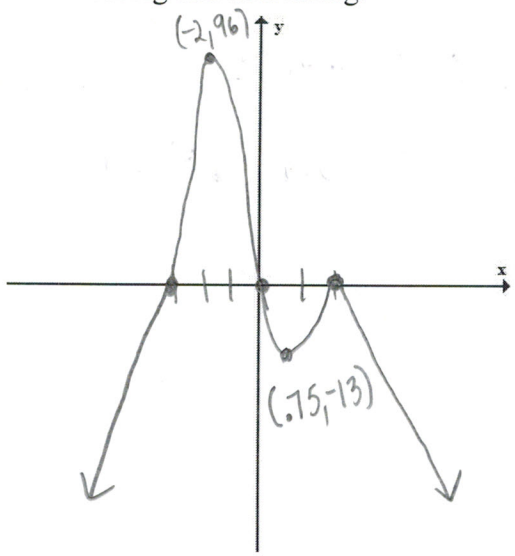
$g(0) = -3(0)(0+3)(0-2)^2$

$= 0$ $\qquad$ $(0,0)$

e. Sketch the graph. Use your graphing utility to find local maximum and local minimum. Label the graph with intercepts and turning points. Identify intervals of increasing and decreasing.

INC: $(-\infty, -2) \cup (.75, 2)$

DEC: $(-2, .75) \cup (2, \infty)$



3. $h(x)=-2x(x^2+4)(x-2)^2$

a. Determine the end behavior of the following polynomial functions by stating the degree and the leading coefficient. Choose one of the four possible end behaviors.

Degree	Leading Coefficient	End Behavior	Max Turning Points
5	-2	↖ ↘ power function $y=-2x^5$	4

b. Complete the following statements.

$f(x)\rightarrow$ $-\infty$ as $x\rightarrow\infty$ $\qquad$ $f(x)\rightarrow$ ∞ as $x\rightarrow-\infty$

c. Find the real zeros for each of the above functions, state the multiplicity of each zero, and how you would graph the function near the zero.

Factor	x	x-2	x+2i	x-2i	
Real Zeros	0	2	none	none	
Multiplicity	1	2			
Cross/Touch	cross	touch			

d. Find the y-intercept.

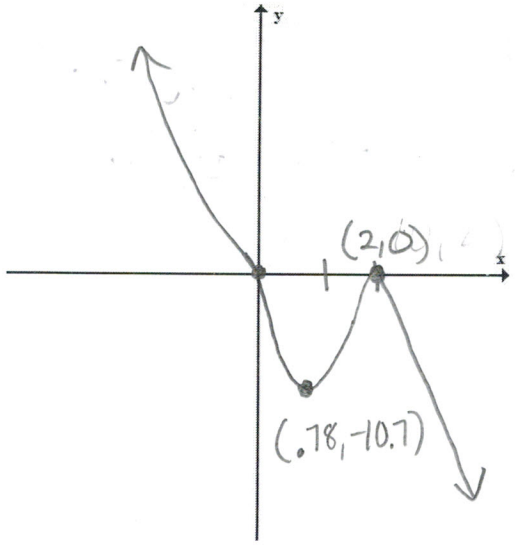
$h(0)=-2(0)(0^2+4)(0-2)^2$

$=0$

e. Sketch the graph. Use your graphing utility to find local maximum and local minimum. Label the graph with intercepts and turning points. Identify intervals of increasing and decreasing.

INC: $(.78,2)$

DEC: $(-\infty,.78)\cup(2,\infty)$



4. $p(x)=6x^2-x-15$

a. Determine the end behavior of the following polynomial functions by stating the degree and the leading coefficient. Choose one of the four possible end behaviors.

Degree	Leading Coefficient	End Behavior	Max Turning Points
2	6	↑ ↑	1

b. Complete the following statements.

$f(x)\to$ ∞ as $x\to\infty$ $\qquad$ $f(x)\to$ ∞ as $x\to-\infty$

c. Find the real zeros for each of the above functions, state the multiplicity of each zero, and how you would graph the function near the zero.

Factor	$2x+3$	$3x-5$			
Real Zeros	$-\frac{3}{2}$	$5/3$			
Multiplicity	1	1			
Cross/Touch	cross	cross			

$P(x)=6x^2-x-15$

$=(2x+3)(3x-5)$

$2x+3=0$

$-3\ \ -3$

$\frac{2x}{2}=\frac{-3}{2}$

$x=-\frac{3}{2}$

$3x-5=0$

$+5\ \ +5$

$\frac{3x}{3}=\frac{5}{3}$

$x=\frac{5}{3}$

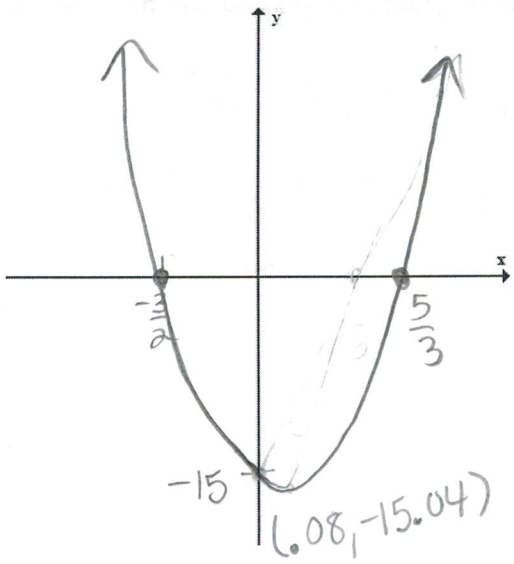
d. Find the y-intercept.

$P(0)=6(0)^2-(0)-15=-15$

e. Sketch the graph. Use your graphing utility to find local maximum and local minimum. Label the graph with intercepts and turning points. Identify intervals of increasing and decreasing.

INC: $(.08,\infty)$

DEC: $(-\infty,.08)$



5. Identify which of the following equations represents a polynomial function. For those that are polynomial functions state the degree and for those that are not polynomial functions state why not.

a. $f(x)=3x^5-3x^4+2x^3-x+9$ poly degree 5

b. $g(x)=\sqrt{x-9}$ not poly square root

c. $h(x)=\frac{1}{2}x^2-\frac{\sqrt{3}}{2}x+\frac{7}{8}$ poly degree 2

d. $F(x)=\frac{3x-9}{9-x^2}$ not poly beause var in denomin

e. $G(x)=3x^{-1}-2x^2+2$ not poly because of neg

f. $H(x)=-2x(x-9)^2(x+5)$ poly degree = 4

g. $h(t)=x^{3/2}-2x-1$ not poly frac power

A not poly corner

B poly ① b) 1 turning point c) least degree = 2

D poly ① ② b) 2TP c) least degree 3

D not poly cusp

E poly ① ② ③ ④ b) 3 TP c) least degree = 4

F not poly hole

G poly ① ② b) 2 TP c) least degree 3

H poly b) 0 TP c) least degree = 1

I not poly gap

↗ Smooth & continuous

For the above graphs:

a. Identify which of the following graphs could represent a polynomial function. If the graph does not represent a polynomial function, state why.

b. For those that could be polynomial functions, how many turning points does the graph have?

c. For those that could be polynomial functions, state the least degree the polynomial function could have.