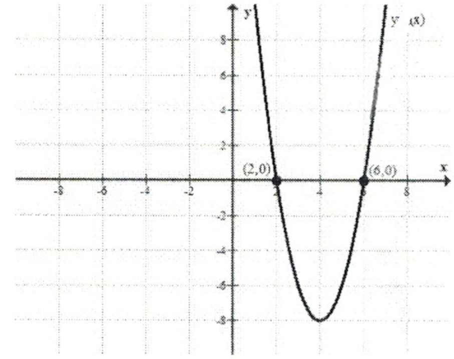
4.5 Practice Problems

1. Use the figure to solve the inequality.



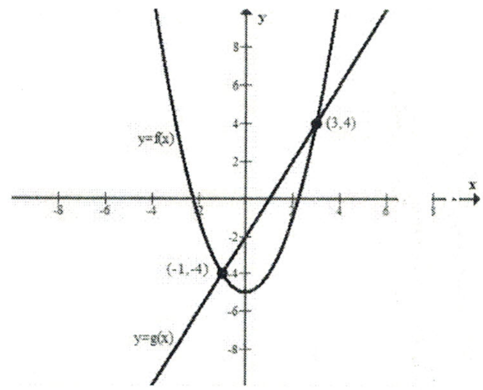
   a. $f(x) \geq 0$

   $(-\infty, 2] \cup [6, \infty)$

   b. $f(x) < 0$

   $(2, 6)$

2. Use the figure to solve the inequality.



   a. $f(x) \geq g(x)$

   $(-\infty, -1] \cup [3, \infty)$

   b. $f(x) < g(x)$

   $(-1, 3)$

3. Solve the inequalities by graphing.

   $x^2 \geq 25$

   $-25 \quad -25$

   $x^2 - 25 \geq 0$

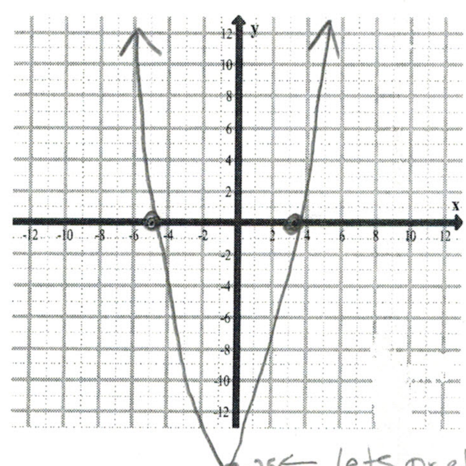
   Graph $y = x^2 - 25$

   x-int $(5,0)$ $(-5,0)$

   y-int $(0,-25)$

Solution to $x^2 \geq 25$

$(-\infty, -5] \cup [5, \infty)$



$-25 \leftarrow$ lets pretend -25 is here

4. Solve the inequality.

   $(x-5)(x+2) < 0$

$x - 5 = 0 \qquad x + 2 = 0$

$x = 5 \qquad x = -2$

$+ \qquad - \qquad +$

$-3 \qquad 0 \qquad 6$

$(-\infty, -2) \quad -2 \quad (-2, 5) \quad 5 \quad (5, \infty)$

pick testpoint in each interval

plug them into $(x-5)(x+2)$

$x = -3 \quad (-3-5)(-3+2) = (-8)(-1) = 8$ pos

$x = 0 \quad (0-5)(0+2) = (-5)(2) = -10$ neg

$x = 6 \quad (6-5)(6+2) = (1)(8) = 8$ pos

Since < 0 solution is neg se

$(-2, 5)$

5. Solve the inequality
$x^2 \geqslant x + 12$

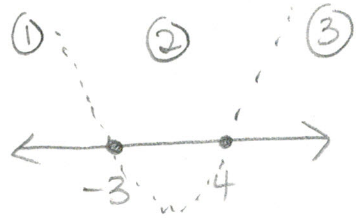
$x^2 - x - 12 \geq 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x - 4 = 0 \quad x + 3 = 0$

$x = 4 \qquad x = -3$



Since $y = x^2 - x - 12$ is an up facing parabola w/ x-int at -3 & 4 we can sketch.

① y-values in the interval $(-\infty, -3)$ are positive

② y-values in the interval $(-3, 4)$ are negative

③ y-values in the interval $(4, \infty)$ are positive

Since ≥ 0 the

$\boxed{\text{Solution is } (-\infty, -3] \cup [4, \infty)}$

6. Solve the inequality.
$2x^2 + 9x + 4 < 0$

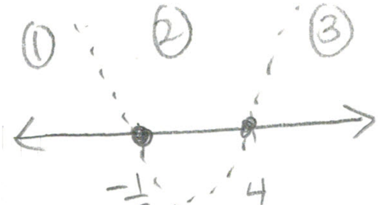
$2x^2 + 9x + 4 = 0$

$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(4)}}{2(2)}$

$= \frac{-9 \pm \sqrt{81 - 32}}{4}$

$= \frac{-9 \pm \sqrt{49}}{4}$

$= \frac{-9 \pm 7}{4} = \begin{cases} \frac{-9+7}{4} = \frac{-2}{4} = -\frac{1}{2} \\ \frac{-9-7}{4} = \frac{-16}{4} = 4 \end{cases}$



Since $y = 2x^2 + 9x + 4$ is an up facing parabola w/ x-int at $-\frac{1}{4}$ & 4 we can sketch

① the interval $(-\infty, -\frac{1}{2})$ has pos y-values

② the interval $(-\frac{1}{2}, 4)$ has neg y-values

③ the interval $(4, \infty)$ has pos y-values

Since < 0

$\boxed{\text{Solution is } (-\tfrac{1}{2}, 4)}$

7. A person standing close to the edge on the top of a 96 foot building throws a baseball vertically upward. The quadratic function

$$s(t) = -16t^2 + 16t + 96$$

models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

a. For what time is the ball more than 48 feet above the ground.

$s(t) > 48$

$-16t^2 + 16t + 96 > 48$
$\qquad\qquad\quad -48 \;\; -48$

$-16t^2 + 16t + 48 > 0$

Solve $-16t^2 + 16t + 48 = 0$

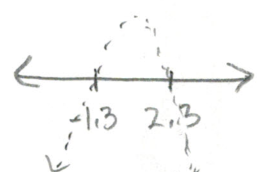
$t^2 - t - 3 = 0$

$t^2 - t - 3 = 0$

$t = \frac{-(-1) \pm \sqrt{1^2 - 4(1)(3)}}{2(1)}$

$= \frac{1 \pm \sqrt{1 + 12}}{2}$

$= \frac{1 \pm \sqrt{13}}{2} \begin{cases} \frac{1+\sqrt{13}}{2} \approx 2.3 \\ \frac{1-\sqrt{13}}{2} \approx -1.3 \end{cases}$



$\boxed{\text{Solution } 0 \leq t < 2.3}$

we adjust this to 0 since time isn't negative

b. For what time is the ball more than 80 feet above the ground.

$s(t) > 80$

$-16t^2 + 16t + 96 > 80$
$\qquad\qquad\quad -80 \;\; -80$

$-16t^2 + 16t + 16 > 0$

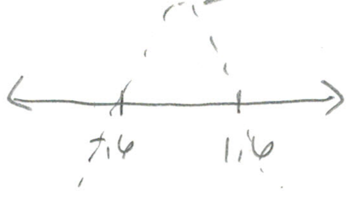
Solve $-16t^2 + 16t + 16 = 0$

$t^2 - t - 1 = 0$

$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$t = \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1 \pm \sqrt{5}}{2} \begin{cases} \frac{1+\sqrt{5}}{2} \approx 1.6 \\ \frac{1-\sqrt{5}}{2} \approx -.6 \end{cases}$



$\boxed{\text{Solution } 0 \leq t < 1.6}$