4.4 Practice Problems

1. The marketing department at a electronics company has found that, when certain tablets are sold at a price of p dollars per unit, the number x of tablets cold is given by the demand equation

$$x = 35000 - 100p$$

a. Find a model that expresses the revenue R as a function of the price p.
b. What is the domain of R?
c. What unit price should be used to maximize revenue?
d. If this price is charged, what is the maximum revenue?
e. How many units are sold at this price?
f. Graph R.
g. What price should the electronics company charge to collect at least \$2,000,000 in revenue?

a) $R = xp$
$= (35000 - 100p)p$
$= 35000p - 100p^2$
$\boxed{R(p) = -100p^2 + 35000p}$

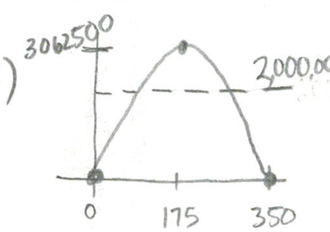
b) $p > 0$
$x \geq 0 \Rightarrow$
$35000 - 100p \geq 0$
$\frac{-100p}{-100} \geq \frac{-35000}{-100}$
$p \leq 350$

$\boxed{\text{Domain } 0 < p \leq 350}$

c) $p = \frac{-b}{2a} = \frac{-35000}{2(-100)} = 175$
price to max revenue is \$175

d) $R(175) = -100(175)^2 + 35000(175)$
$= -3062500 + 6125000$
$= \$3062500$
Maximum revenue is 3,062,500

e) $x = 35000 - 100(175)$
$= 35000 - 17500$
$= 17500$

f)


g) using calculator
$72 \leq p \leq 278$

2. You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

1) Since the fencing goes around the rectangle 80 is the perimeter
$80 = 2l + 2w$
Solve for one variable
$80 = 2l + 2w$
$-2l \quad -2l$
$\frac{80 - 2l}{2} = \frac{2w}{2}$
$40 - l = w$

2) The question asks to maximize Area
$\text{Area} = lw$
$= l(40 - l)$
$A = 40l - l^2$
$A = -l^2 + 40l$

3) Find the vertex
$l = \frac{-b}{2a} = \frac{-40}{2(-1)} = \frac{-40}{-2} = 20$

4) dimensions $l = 20$ yards
$w = 40 - l$
$= 40 - 20$
$= 20$ yards

5) Maximum Area
$A = lw$
$= (20)(20)$
$= 400 \text{ yd}^2$

3. A person standing close to the edge on the top of a 96 foot building throws a baseball vertically upward. The quadratic function

$$s(t)=-16t^2+16t+96$$

models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

a. After how many seconds does the ball reach its maximum height? What is the maxim...m height?

To maximize height find the vertex

$t=\frac{-b}{2a}=\frac{-16}{2(-16)}=.5$ seconds

maximum height occurs .5 seconds after the ball is thrown

The max height is found by plugging into the function

$S(.5)=-16(.5)^2+16(.5)+96$

$=100$ feet

b. How many seconds does it take until the ball finally hits the ground?

when the ball hits the ground the height will be zero.

$S(t)=0$

$-16t^2+16t+96=0$

$t^2-t-6=0$

$(t+2)(t-3)=0$

$t+2=0 \quad t-3=0$

$t=-2 \quad t=3$

disregard the negative solution

the ball hits the ground 3 seconds after it is thrown

c. Find $s(0)$ and describe what this means.

$S(0)=-16(0)^2+16(0)+96$

$=96$ ft

96 is the initial height of the ball