

MAC1105 College Algebra
7.5 Practice Problems

Solve the following systems of nonlinear equations by substitution or elimination.

Rewrite by lining up the like terms & lining up the "="

1. $\begin{cases} x-y=-1 \\ y=x^2+1 \end{cases}$

Back sub
 $x=0$

Back sub
 $x=1$

$x - (x^2+1) = -1$

$y = 0^2+1 = 1$

$y = 1^2+1 = 2$

$x - x^2 - 1 = -1$

$(0, 1)$

$(1, 2)$

$-x^2 + x - 1 = -1$
+1 +1

$-x^2 + x = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x=0 \quad x-1=0$
 $x=1$

3. $\begin{cases} 3x^2+y^2=12 \\ x^2+y^2=4 \end{cases}$

Solutions
 $(0, 1) (1, 2)$

$3x^2 + y^2 = 12$

$-x^2 - y^2 = -4$

$\frac{2x^2}{2} = \frac{8}{2}$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

Back sub
 $x = \pm 2$

$(\pm 2)^2 + y^2 = 4$

$4 + y^2 = 4$

$y^2 = 0$

$y = 0$

Solutions
 $(2, 0) (-2, 0)$

2. $\begin{cases} x^2+y^2=16 \\ y=x^2-4 \end{cases}$

$y^2 + y = 12$

$y^2 + y - 12 = 0$

$(y+4)(y-3) = 0$

$y+4=0 \quad y-3=0$

$y=-4 \quad y=3$

Back sub

$y = -4$

$-4 = x^2 - 4$

$+4 \quad +4$

$x^2 = 25$
 $y = x$

$\begin{cases} x^2+y^2=16 \\ -x^2+y=-4 \end{cases}$

Back sub
 $y=3$

$3 = x^2 - 4$
 $+4 \quad +4$

$7 = x^2$

$\sqrt{7} = \sqrt{x^2}$

$0 = x$

$\pm\sqrt{7} = x$

Solutions
 $(0, -4) (\sqrt{7}, 3) (-\sqrt{7}, 3)$

4. $\begin{cases} xy=25 \\ y=x \end{cases}$

$x \cdot x = 25$

$x^2 = 25$

$x = \pm 5$

Back sub

$x=5$

$x=-5$

$y=5$

$y=-5$

Solutions
 $(5, 5) (-5, 5)$

5. $\begin{cases} 3x^2-2y^2=-5 \\ 2x^2-y^2=-2 \end{cases}$ ← mult by 2

Backsub
 $x = \pm 1$

$3x^2 - 2y^2 = -5$

$-4x^2 + 2y^2 = 4$

$\frac{-x^2}{-1} = \frac{-1}{-1}$

$x^2 = 1$

$x = \pm 1$

$3(\pm 1)^2 - 2y^2 = -5$

$3 - 2y^2 = -5$
 $-3 \quad -3$

$\frac{-2y^2}{-2} = \frac{-8}{-2}$

$y^2 = 4$

$y = \pm 2$

Solutions

$(1, 2) (1, -2)$

$(-1, 2) (-1, -2)$

6. $\begin{cases} (x+3)^2 + (y+4)^2 = 4 \\ y = x-3 \end{cases}$

$(x+3)^2 + (x+3+4)^2 = 4$

$(x+3)^2 + (x+7)^2 = 4$

$x^2 + 6x + 9 + x^2 + 14x + 49 = 4$

$2x^2 + 20x + 58 = 4$

$2x^2 + 20x + 54 = 0$

$2(x^2 + 10x + 27) = 0$

$2(x+3)(x+9) = 0$

$x+3=0 \quad x+9=0$

$x=-3 \quad x=-9$

$(-3, -6) (-9, -12)$

$x=3$

$x=-1$

$y=3-3$

$y=-1-3$

$y=-6$

$y=-4$

$(-3, -6)$

$(-1, -4)$

Solutions

$(-3, -6) (-1, -4)$

$$7. \begin{cases} y = x^2 - 6x + 5 \\ 2x + y = 1 \end{cases}$$

$$2x + x^2 - 6x + 5 = 1$$

$$x^2 - 4x + 5 = 1$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x-2 = 0$$

$$x = 2$$

Back sub

$$2(2) + y = 1$$

$$4 + y = 1$$

$$y = -3$$

$$(2, -3)$$

$$9. \begin{cases} y = x^2 \\ x^2 + y^2 = 6 \end{cases}$$

$$y + y^2 = 6$$

$$y^2 + y - 6 = 0$$

$$(y-2)(y+3) = 0$$

$$y-2 = 0 \quad y+3 = 0$$

$$y = 2 \quad y = -3$$

$$2 = x^2$$

$$-3 = x^2$$

$$\pm\sqrt{2} = x$$

No solution

$$(\sqrt{2}, 2)$$

$$(-\sqrt{2}, 2)$$

$$8. \begin{cases} 2x^2 + y^2 = 17 \\ x^2 + y^2 = 13 \end{cases}$$

$$2x^2 + y^2 = 17$$

$$-x^2 - y^2 = -13$$

$$x^2 = 4$$

$$x = \pm 2$$

Back sub $x = \pm 2$

$$(\pm 2)^2 + y^2 = 13$$

$$4 + y^2 = 13$$

$$y^2 = 9$$

$$y = \pm 3$$

$$(2, 3) (2, -3)$$

$$(-2, 3) (-2, -3)$$

$$10. \begin{cases} y = x \\ x^2 + y^2 = 8 \end{cases}$$

$$x^2 + x^2 = 8$$

$$\frac{2x^2}{2} = \frac{8}{2}$$

$$x^2 = 4$$

$$x = \pm 2$$

$$(2, 2)$$

$$(-2, 2)$$