MAC1105 College Algebra
5.5 Practice Problems

$A = Pe^{rt}$

1. Suppose that a bank offers you an account that pays 5% annually compounded continuously. If you plan to deposit $500, how long will it take for you money to double? Round your answer to the nearest year.

$P = 500$
$r = .05$
$t = ?$
$A = 2(500) = 1000$

$\frac{1000}{500} = \frac{500e^{.05t}}{500}$
$2 = e^{.05t}$
$\ln 2 = \ln e^{.05t}$

$\frac{\ln 2}{.05} = \frac{.05t}{.05}$
$\frac{\ln 2}{.05} = t$

$t \approx 13.86$
14 years to double

2. Suppose that broker tells you that it will take 8 years to double your money on an investment that is compounded continuously. What is the annual interest rate that you will be earning? Round your answer to three decimal places.

$A = Pe^{rt}$

$t = 8$
$r = ?$
$P = P$
$A = 2P$

$\frac{2P}{P} = \frac{Pe^{8r}}{P}$
$2 = e^{8r}$
$\ln 2 = \ln e^{8r}$

$\frac{\ln 2}{8} = \frac{8r}{8}$
$.08664 = r$
$8.67\% = r$

8.67% to double in 8 years

3. The size P of a certain insect population at time (in days) obeys the function $P(t) = 300e^{.05t}$.
a. Determine the number of insects at $t = 0$ days.
b. What is the growth rate of the insect population?
c. Graph the function using the graphing utility.
d. What is the population after 10 days?
e. When will the insect population reach 1120?
f. When will the insect population double?

d. $P(10) = 300e^{.05(10)}$
$= 494.616$
$= 495$

f. $\frac{600}{300} = \frac{300e^{.05t}}{300}$
$2 = e^{.05t}$
$\ln 2 = \ln e^{.05t}$
$\frac{\ln 2}{.05} = \frac{.05t}{.05}$
$13.86 = t$
13.86 days to double

a. $P(0) = 300e^{.05(0)}$
$= 300e^0$
$= 300(1)$
$= 300$

b. $K = .05$

e. $\frac{1120}{300} = \frac{300e^{.05t}}{300}$
$\frac{1120}{300} = e^{.05t}$
$\ln\left(\frac{1120}{300}\right) = \ln e^{.05t}$
$\ln\left(\frac{1120}{300}\right) = .05t$

$\frac{\ln\left(\frac{1120}{300}\right)}{.05} = \frac{.05t}{.05}$
$t = \frac{\ln\left(\frac{1120}{300}\right)}{.05}$
≈ 26.346
26 days

4. The population of Sarnun follows the exponential law. If the population doubled in size over 15 months and he current population is 10,000, what will the population be in 4 years from now?

4 years = 48 months

$P = P_0 e^{Kt}$

doubled in 15 months to find K.

$\frac{2P_0}{P_0} = \frac{P_0 e^{15K}}{P_0}$
$2 = e^{15K}$
$\ln 2 = \ln e^{15K}$
$\frac{\ln 2}{15} = \frac{15K}{15}$
$\frac{\ln 2}{15} = K$
$0.046 = K \leftarrow$ growth rate

$P = 10000e^{\left(\frac{\ln 2}{15}\right)48}$
$P = 91895.868$
$= 91{,}896 \rightarrow$ population after 4 years

use half life of
18 days to find K

5. The chemical element californium has a half-life of approximately 18 days. If 100 grams are present now, how much will be present in 60 days?

$P = P_0 e^{kt}$

$\frac{\frac{1}{2}P_0}{P_0} = \frac{P_0 e^{18K}}{P_0}$

$\frac{1}{2} = e^{18K}$

$\ln(\frac{1}{2}) = \ln e^{18K}$

$\ln(\frac{1}{2}) = 18K$

$\frac{\ln(\frac{1}{2})}{18} = \frac{18K}{18}$

$-0.0385 = K$

$P = 100e^{\frac{\ln(\frac{1}{2})}{18} \cdot 60}$

$= 9.92$

9.92 grams remain after 60 days

6. After the release of a radioactive material into the atmosphere from a nuclear power plant in a country in 1992, the hay in that country was contaminated by a radioactive isotope (half-life 8 days). If it is save the feed the hay to the cows when 10% of the radioactive isotopes remains, how long did the farmers need to wait to use this hay?

half life 8 days to find K

$P = P_0 e^{kt}$

$\frac{\frac{1}{2}P_0}{P_0} = \frac{P_0 e^{8K}}{P_0}$

$\frac{1}{2} = e^{8K}$

$\ln(\frac{1}{2}) = \ln e^{8K}$

$\frac{\ln(\frac{1}{2})}{8} = \frac{8K}{8}$

$K = -0.0866$

I used $\frac{\ln(\frac{1}{2})}{8}$ in this calculation istead of -0.0866 $\longrightarrow$

10% remaining

$\frac{.10P_0}{P_0} = \frac{P_0 e^{-0.0866t}}{P_0}$

$.10 = e^{-0.087t}$

$\ln(.10) = \ln e^{-.087t}$

$\frac{\ln(.10)}{-.087} = \frac{-.087t}{-.087}$

$26.58 = t$

27 days t

7. The amount of caffeine in the human body has a half life of 4.9 hours to 6 hours depending on the person.

a. Assuming the caffeine half life for a particular person is 5 hours find the decay rate of caffeine and build an exponential model $\left[A = A_0 e^{kt}\right]$ for the decay rate. Round your answer to three decimal places.

b. Using the model in part a, find the amount of caffeine left in the body 1.5 hours after drinking a can of coke which contains 39.5 grams of caffeine. Round your answer to three decimal places.

a. half life of 5 days

$\frac{\frac{1}{2}A_0}{A_0} = \frac{A_0 e^{5K}}{A_0}$

$\frac{1}{2} = e^{5K}$

$\ln(\frac{1}{2}) = \ln e^{5K}$

$\frac{\ln(\frac{1}{2})}{5} = \frac{5K}{5}$

$\frac{\ln(\frac{1}{2})}{5} = K$

$-0.139 = K$

b. $A = 39.5e^{-.139 \cdot 1.5}$

$= 32.06$

32.06 grams remaining 1.5 hours after drinking