

MAC1105 College Algebra
5.3 Practice Problems

In problems 1 - 4, use the properties of logarithms to expand each logarithmic expression as much possible. Where possible, evaluate logarithmic expressions without using a calculator.

$$\begin{aligned} 1. \log(10000xy) \\ &= \log 10000 + \log x + \log y \\ &= 4 + \log x + \log y \end{aligned}$$

$$\begin{aligned} 2. \log_3 \frac{81}{x} \\ &= \log_3 81 - \log_3 x \\ &= 4 - \log_3 x \end{aligned}$$

$$\begin{aligned} 3. \ln \frac{e^3}{x} \\ &= \ln e^3 - \ln x \\ &= 3 \ln e - \ln x \\ &= 3(1) - \ln x \\ &= 3 - \ln x \end{aligned}$$

$$\begin{aligned} 4. \log_4 \frac{16x^2}{y^3} \\ &= \log_4 16 + \log_4 x^2 - \log_4 y^3 \\ &= 2 + 2 \log_4 x - 3 \log_4 y \end{aligned}$$

In problems 5 - 8, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

$$\begin{aligned} 5. \log 25 + \log 4 \\ &= \log(100) \\ &= 2 \end{aligned}$$

$$\begin{aligned} 6. \log_2 x - \log_2 y \\ &= \log_2 \left(\frac{x}{y} \right) \end{aligned}$$

$$\begin{aligned} 7. 2 \ln x + 4 \ln y - 3 \ln z \\ &= \ln x^2 + \ln y^4 - \ln z^3 \\ &= \ln \left(\frac{x^2 y^4}{z^3} \right) \end{aligned}$$

$$\begin{aligned} 8. 2 \log_3 x - 3 \log_3 y \\ &= \log_3 x^2 - \log_3 y^3 \\ &= \log_3 \left(\frac{x^2}{y^3} \right) \end{aligned}$$

In problems 9 - 10, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places. (Use the change of base formula.)

9. $\log_8 25$

$$= \frac{\log 25}{\log 8} \quad \text{or} \quad = \frac{\ln 25}{\ln 8}$$

$$\approx 1.55$$

10. $\log_{27} 13$

$$\frac{\log 13}{\log 27} \quad \text{or} \quad \frac{\ln 13}{\ln 27}$$

$$\approx 0.78$$

Solving the following logarithmic equations. Use property of equality

11. $\log_2(x - 5) = \log_2 17$

$$\begin{aligned} x - 5 &= 17 \\ +5 &+5 \\ x &= 22 \end{aligned}$$

Check
 $\log_2(22 - 5) = \log_2(17)$
 ✓

12. $\log(x - 9) = \log(x + 4) + \log 3$

$$\log(x - 9) = \log[3(x + 4)]$$

$$\begin{aligned} x - 9 &= 3x + 12 \\ -x &\quad -x \\ -9 &= 2x + 12 \\ -12 &\quad -12 \\ -21 &= \frac{2x}{2} \quad x = \frac{-21}{2} \end{aligned}$$

Check
 $\log(\frac{-21}{2} - 9)$
 can't take
 log of
 negative

No solution

13. $2\log_7 x = \log_7 64$

power rule: $\log_7 x^2 = \log_7 64$

$$\begin{aligned} x^2 &= 64 \\ \sqrt{x^2} &= \sqrt{64} \\ x &= \pm 8 \end{aligned} \quad \boxed{x=8}$$

Check
 $2\log_7 8 \text{ OK}$ $2\log_7(-8) \leftarrow \text{no}$

15. $\log(x + 3) + \log(x - 2) = \log 14$

condense $\log[(x + 3)(x - 2)] = \log 14$

$$\begin{aligned} (x + 3)(x - 2) &= 14 \\ x^2 - 2x + 3x - 6 &= 14 \\ x^2 + x - 20 &= 0 \\ (x + 5)(x - 4) &= 0 \\ x + 5 = 0 &\quad x - 4 = 0 \\ \cancel{x = -5} &\quad x = 4 \end{aligned}$$

14. ~~$\log(x - 9) = \log(x + 4) + \log 3$~~

check:

$$x = -5 \quad \log(-5 + 3) = \log(-2) \leftarrow \text{no}$$

$$\begin{aligned} x = 4 \quad \log(4 + 3) &= \log(7) \leftarrow \text{OK} \\ \log(4 - 2) &= \log(2) \leftarrow \text{OK} \end{aligned}$$

$$\boxed{x=4}$$