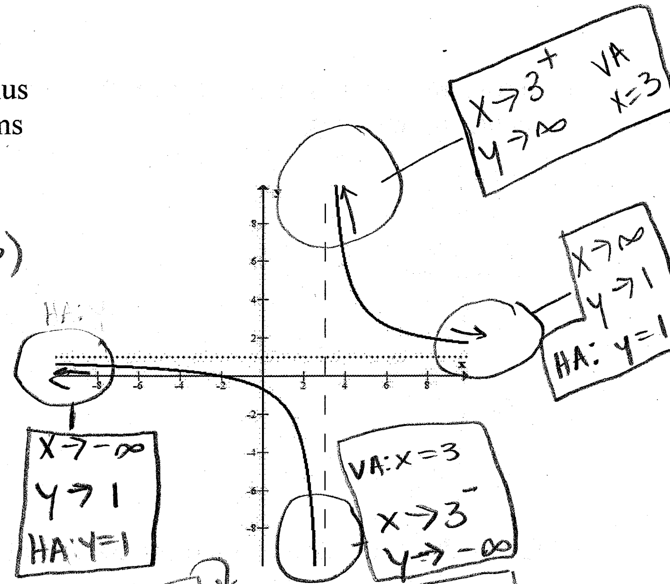
MAC1140 Precalculus
5.4 Practice Problems

1. Use the graph to find the following.



Domain: $(-\infty,3)\cup(3,\infty)$ Range: $(-\infty,1)\cup(1,\infty)$

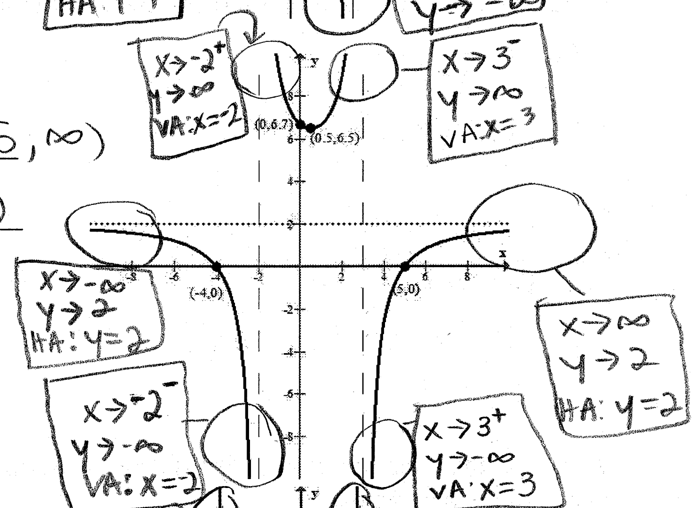
x-intercepts: $(-2,0)$ y-intercepts: $(-1,0)$

Horizontal Asymptote: $y=1$

Vertical Asymptote(s): $x=3$

Oblique/Slant Asymptote: none

2. Use the graph to find the following.



Domain: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$ Range: $(-\infty,2)\cup[6.5,\infty)$

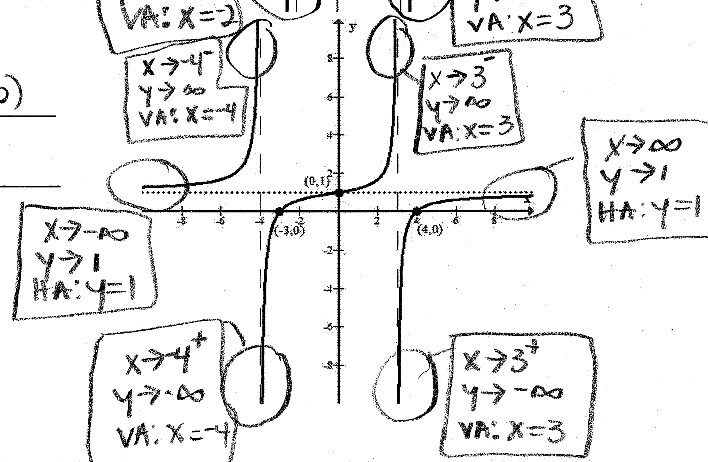
x-intercepts: $(-4,0)$, $(5,0)$ y-intercepts: $(0,6.7)$

Horizontal Asymptote: $y=2$

Vertical Asymptote(s): $x=-2$, $x=3$

Oblique/Slant Asymptote: none

3. Use the graph to find the following.



Domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$ Range: $(-\infty,\infty)$

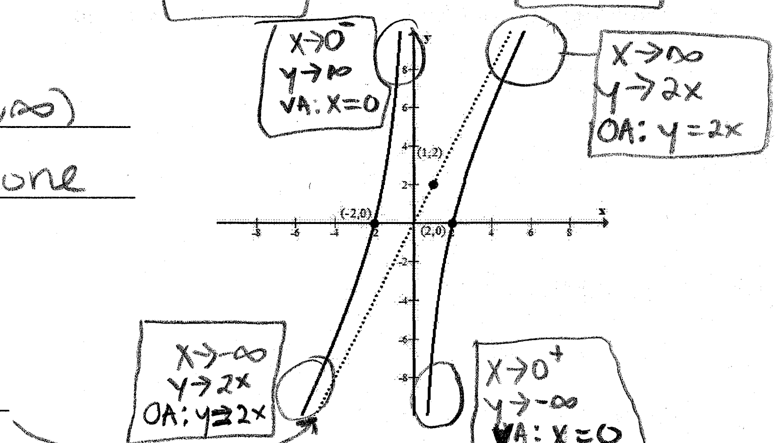
x-intercepts: $(-3,0)$, $(4,0)$ y-intercepts: $(0,1)$

Horizontal Asymptote: $y=1$

Vertical Asymptote(s): $x=-4$, $x=3$

Oblique/Slant Asymptote: none

4. Use the graph to find the following.



Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,\infty)$

x-intercepts: $(-2,0)$, $(2,0)$ y-intercepts: none

Horizontal Asymptote: none

Vertical Asymptote(s): $x=0$

Oblique/Slant Asymptote: $y=2x$

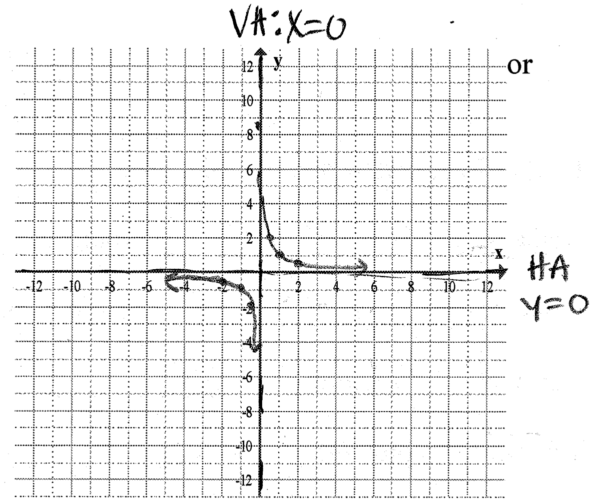
Think about the slope & y-int of the line

5. Graph the base function. Identify the location of any vertical or horizontal asymptotes.

$f(x)=\frac{1}{x}$ called the reciprocal function

x	y
-2	-1/2
-1	-1
-1/2	-2
1/2	2
1	1
2	1/2

$f(-\frac{1}{2})=\frac{1}{-\frac{1}{2}}=1\div-\frac{1}{2}=1\cdot\frac{-2}{1}=-2$



6. Graph the rational function using transformations.

$f(x)=-\frac{1}{x-2}+5$

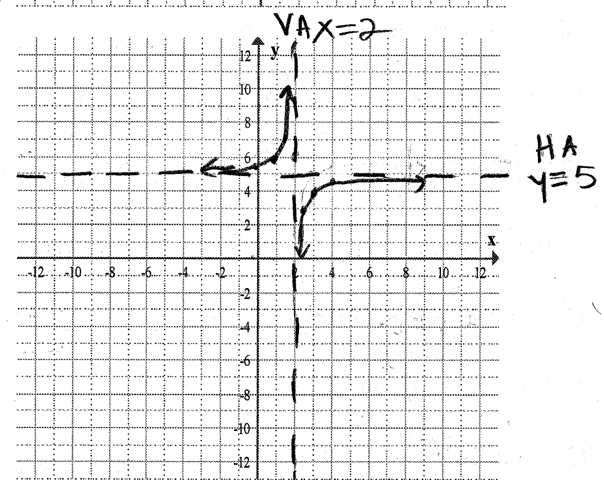
Base $y=\frac{1}{x}$

Transformations

2 − reflection over the x-axis

1 −2 shift right 2 units

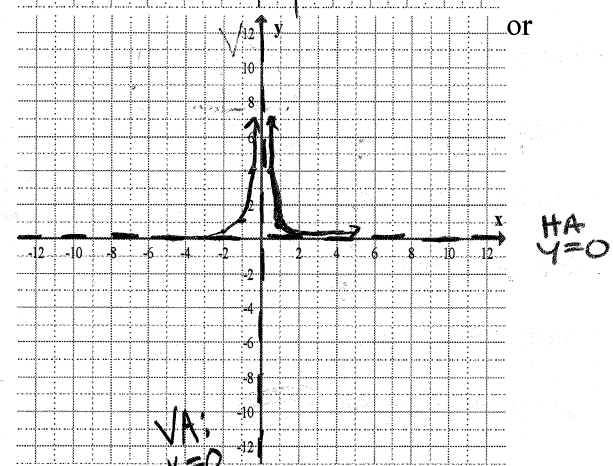
3 5 shift up 5 units



7. Graph the base function. Identify the location of any vertical or horizontal asymptotes.

$f(x)=\frac{1}{x^2}$

x	y
-2	$1/(-2)^2=\frac{1}{4}$
-1	$1/(-1)^2=1$
-1/2	$1/(-\frac{1}{2})^2=1/\frac{1}{4}=4$
1/2	$1/(\frac{1}{2})^2=1/\frac{1}{4}=4$
1	$1/1^2=1$
2	$1/2^2=1/4$



8. Graph the rational function using transformations.

$f(x)=-\frac{1}{(x-2)^2}+5$

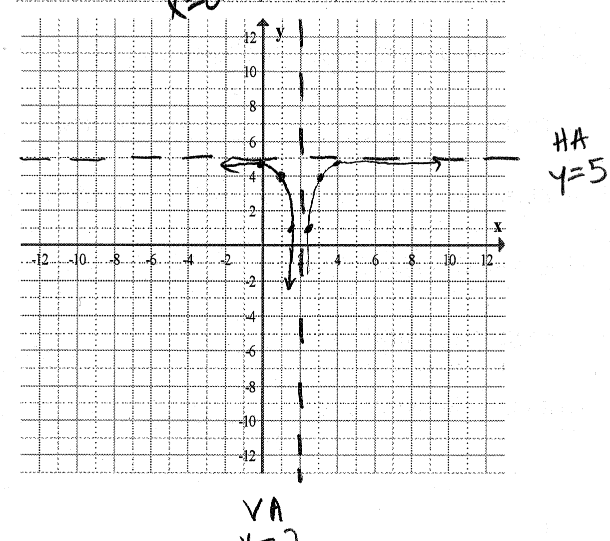
Base: $y=\frac{1}{x^2}$

Transformations

− reflection over the x-axis

−2 shift right 2 units

5 shift up 5 units



9. Find the domain of the following rational functions.

a. $f(x)=\frac{3}{x+2}$

$x+2=0$
$-2\ \ -2$
$x=-2$

D: $(-\infty,-2)\cup(-2,\infty)$

b. $g(x)=\frac{x+1}{2x-6}$ D: $(-\infty,3)\cup(3,\infty)$

$2x-6=0$
$+6\ \ +6$
$\frac{2x}{2}=\frac{6}{2}$
$x=3$

c. $h(x)=\frac{3x^2-6x+2}{x-2}$

$x-2=0$ D: $(-\infty,2)\cup(2,\infty)$
$x=2$

d. $R(x)=\frac{x^2-9}{x^2+x-2}$

$x^2+x-2=0$ D: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
$(x+2)(x-1)=0$
$x+2=0\quad x-1=0$
$x=-2\quad x=1$

e. $G(x)=\frac{x^2+2x-15}{x^2-25}$

$x^2-25=0$
$(x+5)(x-5)=0$
$x+5=0\quad x-5=0$
$x=-5\quad x=5$

D: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

f. $G(x)=\frac{x^4-16}{x^2+9}$

$x^2+9=0$
$x^2=-9$
$\sqrt{x^2}=\sqrt{-9}$
$x=\pm 3i$

D: $(-\infty,\infty)$

10. Find the vertical asymptotes of the function, if any.

a. $f(x)=\frac{3}{x+2}=\frac{3}{x+2}$

$x+2=0$
$x=-2$

VA: $x=-2$

b. $g(x)=\frac{x+1}{2x-6}=\frac{x+1}{2x-6}$

$2x-6=0$
$\frac{2x}{2}=\frac{6}{2}$
$x=3$

VA: $x=3$

c. $h(x)=\frac{3x^2-6x+2}{x-2}=\frac{3x^2-6x+2}{x-2}$

$x-2=0$
$+2\ \ +2$
$x=2$

VA: $x=2$

d. $R(x)=\frac{x^2-9}{x^2+x-2}=\frac{(x+3)(x-3)}{(x+2)(x-1)}$

$(x+2)(x-1)=0$
$x+2=0\quad x-1=0$
$x=-2\quad x=1$

VA: $x=-2,\ x=1$

e. $G(x)=\frac{x^2+2x-15}{x^2-25}=\frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}(x-5)}=\frac{x-3}{x-5}$

$x-5=0$
$x=5$

VA: $x=5$

f. $G(x)=\frac{x^4-16}{x^2+9}=\frac{(x^2-4)(x^2+4)}{x^2+9}=\frac{(x+2)(x-2)(x^2+4)}{x^2+9}$

$x^2+9=0$
$x^2=-9$
$x=\pm 3i$

No VA

11. Use your work from above to decide if the function has a hole. Identify the location of the holes, if any.

a. $f(x)=\frac{3}{x+2}$

No hole

b. $g(x)=\frac{x+1}{2x-6}$

No hole

c. $h(x)=\frac{3x^2-6x+2}{x-2}$

No hole

d. $R(x)=\frac{x^2-9}{x^2+x-2}$

No hole

e. $G(x)=\frac{x^2+2x-15}{x^2-25}=\frac{x-3}{x-5}$

hole @ $x=-5$

$\frac{-5-3}{-5-5}=\frac{-8}{-10}=\frac{4}{5}$

hole $(-5,\frac{4}{5})$

f. $G(x)=\frac{x^4-16}{x^2+9}$

no hole

12. Find the horizontal asymptote or oblique asymptote of the functions, if any.

a. $f(x)=\frac{3}{x+2}$ degree = 0, degree = 1

case 1

HA: $y=0$

b. $g(x)=\frac{1x+1}{2x-6}$ degree=1, degree=1

Case 2

HA: $y=\frac{1}{2}$

c. $h(x)=\frac{3x^2-6x+2}{x-2}$ degree=2, degree=1

Case 3

$x-2\,\overline{)\,3x^2-6x+2}$ quotient $3x$; $3x^2-6x$; $0+2$

OA: $y=3x$

d. $R(x)=\frac{1x^2-9}{1x^2+x-2}$ degree=2, degree=2

Case 2

HA: $y=1$

e. $G(x)=\frac{1x^2+2x-15}{1x^2-25}$ degree2, degree 2

case 2

HA: $y=1$

f. $G(x)=\frac{x^4-16}{x^2+9}$ degree = 4, degree = 2

Case 4

No HA

No OA

13. Find any points for which the graph crosses the horizontal or oblique asymptote.

a. $f(x)=\frac{3}{x+2}$

HA = f(x)

$(x+2)\cdot 0=\frac{3}{\cancel{x+2}}\cdot\cancel{x+2}$

$0=3$.

No Solution

$\boxed{\text{None}}$

b. $g(x)=\frac{x+1}{2x-6}$ HA=g(x)

$\frac{1}{2}=\frac{x+1}{2x-6}$

$1(2x-6)=2(x+1)$

$2x-6=2x+2$

$-2x \quad -2x$

$-6=2$ No Solution

$\boxed{\text{None}}$

c. $h(x)=\frac{3x^2-6x+2}{x-2}$ OA = h(x)

$(x-2)\,3x=\frac{3x^2-6x+2}{\cancel{x-2}}\cdot\cancel{(x-2)}$

$3x^2-6x=3x^2-6x+2$

$-3x^2+6x \quad -3x^2+6x$

$0=2$ No Solution

$\boxed{\text{None}}$

d. $R(x)=\frac{x^2-9}{x^2+x-2}$ HA = R(x)

$(x^2+x-2)1=\frac{x^2-9}{\cancel{x^2+x-2}}\cdot\cancel{(x^2+x-2)}$

$x^2+x-2=x^2-9$

$-x^2 \quad -x^2$

$x-2=-9$

$x-2=-9$

$+2 \quad +2$

$x=-7$

Crosses @ (-7,1)

e. $G(x)=\frac{x^2+2x-15}{x^2-25}$ HA = G(x) Reduced

$1=\frac{x-3}{x-5}$

$x-5=x-3$

$-x \quad -x$

$-5=-3$

No Solution

$\boxed{\text{None}}$

f. $G(x)=\frac{x^4-16}{x^2+9}$

No Asymptote to cross

14. Find the intercepts of the rational functions.

a. $f(x)=\frac{3}{x+2}$

x-int y=0

$\cdot 0=\frac{3}{\cancel{x+2}}\cdot\cancel{x+2}$

$0=3$.

No Solution

No x-int

y-int f(0)

$f(0)=\frac{3}{0+2}=\frac{3}{2}$

y-int $(\frac{3}{2},0)$

b. $g(x)=\frac{x+1}{2x-6}$ If rat is reduced

x-int y=0 or num=0

$x+1=0$

$x=-1$

$(-1,0)$

y-int $(0,-\frac{1}{6})$

$g(0)=\frac{0+1}{2(0)-6}$

$=-\frac{1}{6}$

c. $h(x)=\frac{3x^2-6x+2}{x-2}$

x-int

$3x^2-6x+2=0$

$x=\frac{6\pm\sqrt{(-6)^2-4(3)(2)}}{2(3)}$

$=\frac{6\pm\sqrt{12}}{6}=\frac{6\pm2\sqrt{3}}{6}=\frac{3\pm\sqrt{3}}{3}=$ 1.6, .42

y-int h(0)

$=\frac{3(0)^2-6(0)+2}{0-2}$

$=-1$

$(0,-1)$

d. $R(x)=\frac{x^2-9}{x^2+x-2}$

x-int

$x^2-9=0$

$x^2=9$

$x=\pm3$

(3,0)

(-3,0)

y-int

$R(0)=\frac{0^2-9}{0^2+0-2}$

$=\frac{9}{2}$

$(0,\frac{9}{2})$

e. $G(x)=\frac{x^2+2x-15}{x^2-25}=\frac{x-3}{x-5}$

x-int

$x-3=0$

$x=3$

(3,0)

y-int

$\frac{0-3}{0-5}$

$=\frac{3}{5}$

$(0,\frac{3}{5})$

f. $G(x)=\frac{x^4-16}{x^2+9}=\frac{(x+2)(x-2)(x^2+4)}{x^2+9}$

$(x+2)(x-2)(x^2+4)=0$

$x+2=0 \quad x-2=0 \quad x^2+4=0$

$x=-2 \quad x=2 \quad x^2=-4$

~~$x=\pm2i$~~

x int

(-2,0)

(2,0)

y-int

$\frac{0^4-16}{0^2+9}=-\frac{16}{9}$

≈ -1.8

$\boxed{9\text{-}14\ a}$ $f(x)=\frac{3}{x+2}$

Domain: $(-\infty,-2)\cup(-2,\infty)$

VA: $x=-2$; No hole

HA: $y=0$; No crossing

x-int: None

y-int: $(0,\frac{3}{2}) \to (0,1.5)$

$\boxed{9\text{-}14b}$ $g(x)=\frac{x+1}{2x-6}$

Domain: $(-\infty,3)\cup(3,\infty)$

VA: $x=3$; no hole

HA: $y=\frac{1}{2}$; No crossing

x-int: $(-1,0)$

y-int: $(0,-1/6)$

$\boxed{9\text{-}14c}$ $h(x)=\frac{3x^2-6x+2}{x-2}$

Domain: $(-\infty,2)\cup(2,\infty)$

VA: $x=2$; No hole

HA: None

OA: $y=3x$; No crossing

x-int: $(1.6,0)$ $(.42,0)$

y-int: $(0,-1)$

$\boxed{9\text{-}14\ d}$ $R(x)=\frac{x^2-9}{x^2+x-2}$

Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

VA: $x=-2$, $x=1$; No hole

HA: $y=1$; cross@ $(-7,1)$

x-int: $(3,0)$ $(-3,0)$

y-int: $(0,\frac{9}{2}) \to (0,4.5)$

$\boxed{9\text{-}14e}$ $G(x)=\frac{x^2+2x-15}{x^2-25}$

Domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

VA: $x=5$; hole @ $(-5,\frac{4}{5})$

HA: $y=1$; No crossing

x-int: $(3,0)$

y-int: $(0,\frac{3}{5})$

$\boxed{9\text{-}14f}$ $G(x)=\frac{x^4-16}{x^2+9}$

Domain: $(-\infty,\infty)$

VA: None; No holes

HA: None

OA: None

x-int: $(2,0)$ $(-2,0)$

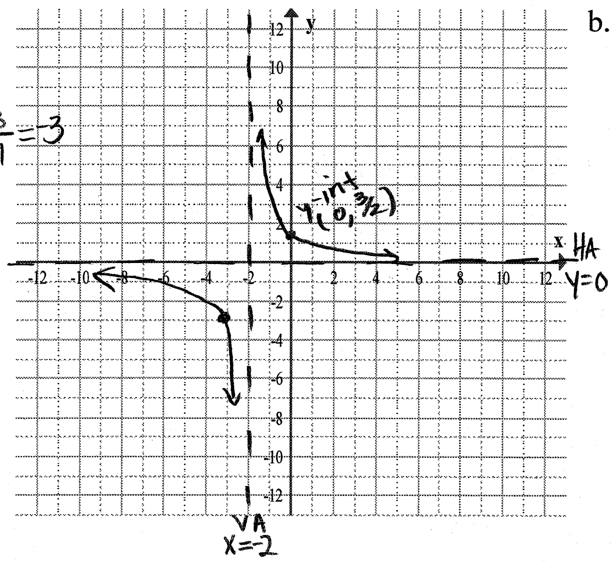
y-int: $(0,-\frac{16}{9}) \to (0,-1.8)$

Summary of info for each function

15. Use the information in 9-14 to graph a-f.

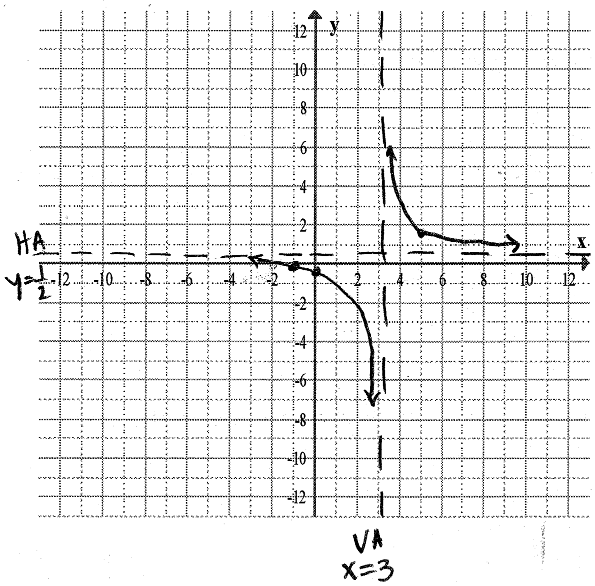
a.

x	y
-3	$\frac{3}{-3+2} = \frac{3}{-1} = -3$



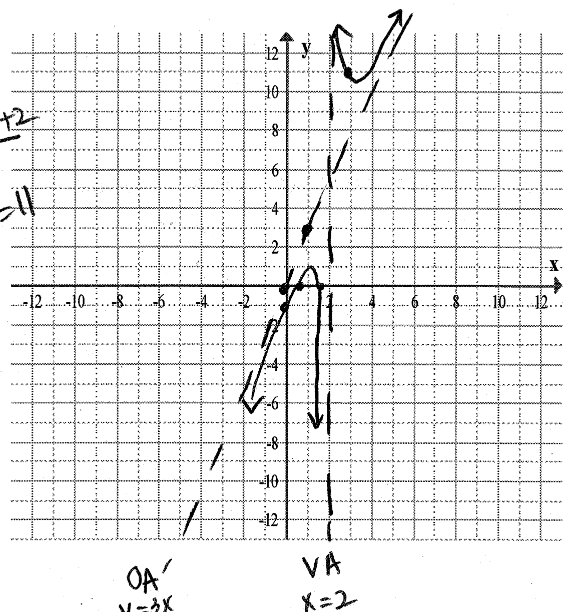
b.

x	y
5	$\frac{5+1}{2(5)-6} = \frac{6}{4} = \frac{3}{2}$

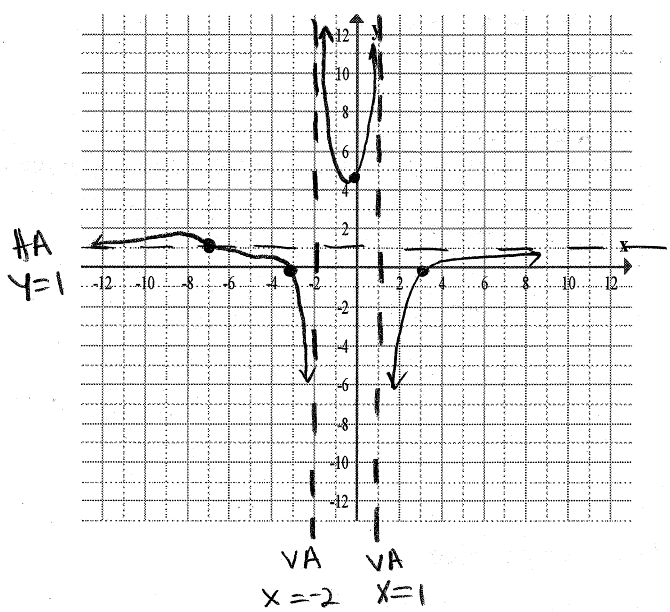


c.

x	y
3	$\frac{3(3)^2-6(3)+2}{3-2} = \frac{11}{1} = 11$

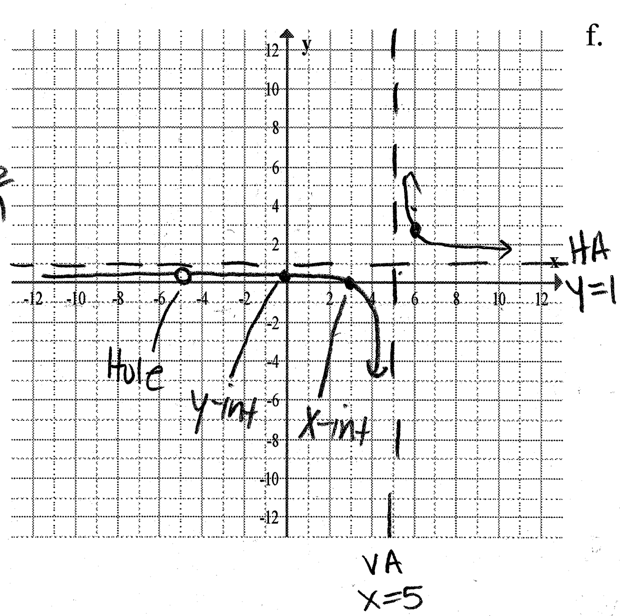


d.

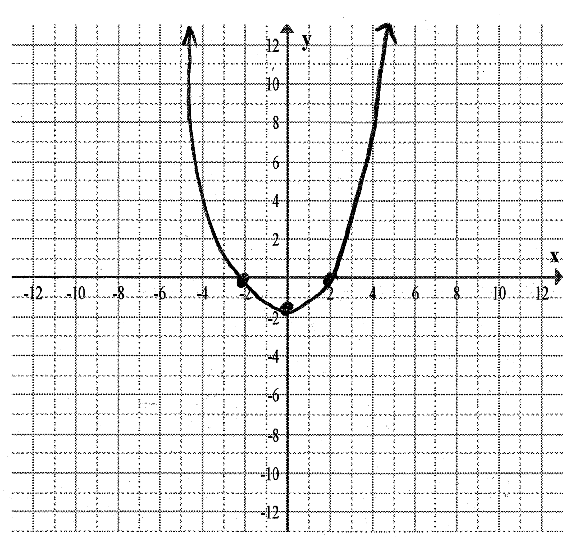


e.

x	y
6	$\frac{6-3}{6-5} = \frac{3}{1} = 3$



f.



Use the function $f(x)=\frac{2x}{x^2-9}$ to answer the following questions.

16. Find the domain of the function.

$x^2-9=0$

$(x+3)(x-3)=0$

$x+3=0$ $x-3=0$

$x=-3$ $x=3$

D: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

17. Find any vertical asymptotes or holes in the graph.

$f(x)=\frac{2x}{(x+3)(x-3)}$

VA: $x=-3$, $x=3$

$(x+3)(x-3)=0$

$x+3=0$ $x-3=0$

$x=-3$ $x=3$

No hole

18. Find any horizontal or oblique asymptotes and any points where the function crosses these asymptotes.

$f(x)=\frac{2x}{x^2-9}$ → Degree 1 → Degree 2

case 1

HA: $y=0$

$0=\frac{2x}{x^2-9}$

$(x^2-9)(0)=\frac{2x}{x^2-9}\cdot x^2-9$

$\frac{0}{2}=\frac{2x}{2}$

$0=x$

Crosses @ (0,0)

19. Find any x or y intercepts.

x-int

$\frac{2x}{2}=\frac{0}{2}$

$x=0$

(0,0)

y-int

$f(0)=\frac{2(0)}{0^2-9}$

$=0$

(0,0)

20. Use the information to graph the function.

Need more points to get a good picture

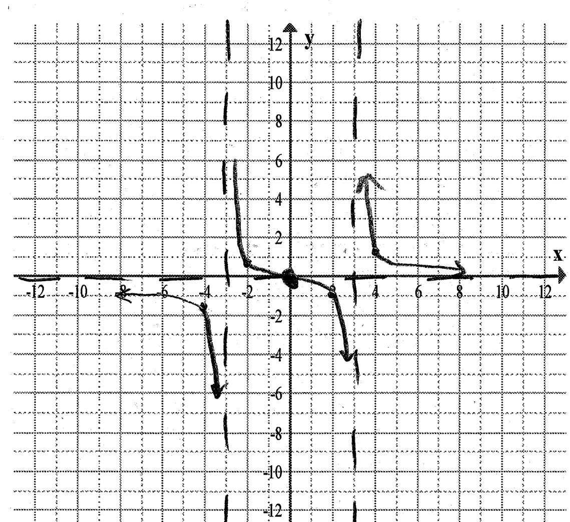
x	y
-4	-1.14
-2	.8
2	-.8
4	1.14

$f(-4)=\frac{2(-4)}{(-4)^2-9}=\frac{-8}{16-9}=\frac{-8}{7}=-1.14$

$f(-2)=\frac{2(-2)}{(-2)^2-9}=\frac{-4}{4-9}=\frac{-4}{-5}=\frac{4}{5}=.8$

$f(2)=\frac{2(2)}{2^2-9}=\frac{4}{4-9}=\frac{4}{-5}=-\frac{4}{5}=-.8$

$f(4)=\frac{2(4)}{}=\frac{8}{}=\frac{8}{}=1.14$



Choose x in each section

$x=-4$ $x=-2$ $x=2$ $x=4$