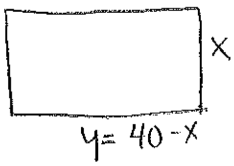
MAC1105 College Algebra

4.2 Practice Problems

1. You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?



$P = 2x + 2y$

$80 = 2x + 2y$

$\frac{80 - 2x}{2} = \frac{2y}{2}$

$40 - x = y$

$A = xy$

$A = x(40-x)$

$A = 40x - x^2$

$A = -x^2 + 40x$

Maximum of a quadratic function occurrs at the vertex

$h = \frac{-b}{2a} = \frac{-40}{2(-1)} = \frac{-40}{-2} = 20$

Maximum area occurs when one side is 20 yards

the other side $y = 40 - 20 = 20$ yards

Maximum Area $= 20 \times 20$
$= 400$ yds

2. A person standing close to the edge on the top of a 96 foot building throws a baseball vertically upward. The quadratic function

$$s(t) = -16t^2 + 16t + 96$$

models the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

a. After how many seconds does the ball reach its maximum height? What is the maximum height?

b. How many seconds does it take until the ball finally hits the ground?

c. Find $s(0)$ and describe what this means.

(a) Max occurs at the vertex

$h = -\frac{b}{2a} = \frac{-16}{2(-16)}$
$= .5$ Seconds

$k = f(.5) = -16(.5)^2 + 16(.5) + 96$
$= 100$ feet

Max height of 100 ft .5 seconds after thrown

(b) hits the ground $\Rightarrow$ height $= 0$
$S(t) = 0$

$0 = -16t^2 + 16t + 96$

$0 = -16(t^2 - t - 6)$

$0 = -16(t-3)(t+2)$

$t - 3 = 0 \quad t + 2 = 0$

$t = 3 \quad \cancel{t = -2}$

The ball hits the ground 3 seconds after thrown

(c) $S(0) = -16(0)^2 + 16(0) + 96$
$= 96$ ft

Since $t = 0$ seconds the height of the ball is 96 ft right before it is thrown

3. The monthly revenue R achieved by selling x baseball gloves is figured to be $R(x) = 80x - 0.5x^2$. The monthly cost C of selling x baseball gloves is $C(x) = 20x + 1000$.

a. How many baseball gloves must the company sell to maximize revenue? What is the maximum revenue? (Round to the nearest integer as needed)

b. Profit is given as $P(x) = R(x) - C(x)$. What is the profit function?

c. How many baseball gloves must the company sell to maximize profit? What is the maximum profit?

a) $R(x) = 80x - 0.5x^2$

Max occurs at the vertex

$h = \frac{-b}{2a} = \frac{-80}{2(-.5)} = 80$

$k = R(80) = 3200$

A max revenoe of 3200 dollars occurs when selling 80 gloves

b) $P(x) = R(x) - C(x)$

$= 80x - .5x^2 - (20x + 1000)$

$= 80x - .5x^2 - 20x - 1000$

$= -.5x^2 + 60x - 1000$

c) Max occurs at the vertex

$h = -\frac{b}{2a} = \frac{-60}{2(-.5)} = 60$

$k = P(60) = 800$

A max profit of \$800 occurs when selling 60 gloves.

4. The marketing department at a electronics company has found that, when certain tablets are sold at a price of p dollars per unit, the number x of tablets cold is given by the demand equation

$$x = 35000 - 100p$$

a. Find a model that expresses the revenue R as a function of the price p.
b. What is the domain of R?
c. What unit price should be used to maximize revenue?
d. If this price is charged, what is the maximum revenue?
e. How many units are sold at this price?

a) $R = xp$
$= (35000 - 100p)p$
$= 35000p - 100p^2$

b) $p > 0$
and
$x \geq 0$
$35000 - 100p \geq 0$
$-100p \geq -35000$
$p \leq 350$

b) $0 < p \leq 350$

c) max occurs at vertex
$h = -\frac{b}{2a} = -\frac{35000}{2(-100)}$
$= 175$

d) $R = R(175)$
$= 35000(175) - 100(175)^2$
$= 3,062,500$

e) $x = 35000 - 100(175)$
$= 35000 - 17500$
$= 17500$

5. A projectile is fired from a cliff 180 feet above the water at an inclination of $45°$ to the horizontal, with a muzzle velocity of 45 feet per second. The height h of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(45)^2} + x + 180$$

where x is the horizontal distance of the projectile from the face of the cliff. Use this information to answer the following.

a. At what horizontal distance from the face of the cliff is the height of the projectile a maximum.
b. Find the maximum height of the projectile.

a) Max occurs at the vertex
$h = \frac{-b}{2a} = \frac{-1}{2\left(\frac{-32}{45^2}\right)} = \frac{45^2}{64} = \frac{2025}{64} \approx 31.6$

b) $k = h(31.6) = \frac{-32(31.6)^2}{45^2} + 31.6 + 180$
$= \frac{-32(998.56)}{2025} + 211.6$
$= -15.80 + 211.6$
$= 195.8$