Solve the linear equations in questions 1-6.

Linear 1. $4x - 5 = 23$

$+5 \quad +5$

$\dfrac{4x}{4} = \dfrac{28}{4}$

Solution
$x = 7$

$x = 7$

Cheek $x = 7$

$4(7) - 5 = 23$

$28 - 5 = 23$

$23 = 23$

True

Linear 2. $2x - 4(5x + 1) = 3x + 17$

$2x - 20x - 4 = 3x + 17$

$-18x - 4 = 3x + 17$

$-3x -3x$

$-21x - 4 = 17$

$+4 \quad +4$

$\dfrac{-21x}{-21} = \dfrac{21}{-21}$

$x = -1$

Solution
$x = -1$

Cueck $x = -1$

$2(-1) - 4(5(-1) + 1) = 3(-1) + 17$

$-2 - 4(-5 + 1) = -3 + 17$

$-2 - 4(-4) = 14$

$-2 + 16 = 14$

$14 = 14$

True

Linear 3. $\dfrac{x}{4} = 2 - \dfrac{x-3}{3}$

use clearing fractions

LCM of 4 & 3 = 12

$12 \cdot \dfrac{x}{4} = 12 \cdot 2 - 12\left(\dfrac{x-3}{3}\right)$

$3x = 24 - 4(x - 3)$

$3x = 24 - 4x + 12$

$3x = 36 - 4x$

$+4x +4x$

$\dfrac{7x}{7} = \dfrac{36}{7}$

$x = \dfrac{36}{7}$

Solution
$x = \dfrac{36}{7}$

Linear 4. $\dfrac{1}{3}(2x - 3) = \dfrac{4}{3}x - \dfrac{x+3}{6}$

LCM = 6

$6 \cdot \dfrac{1}{3}(2x - 3) = 6 \cdot \dfrac{4}{3}x - 6\left(\dfrac{x+3}{6}\right)$

$2(2x - 3) = 2 \cdot (4x) - (x + 3)$

$4x - 6 = 8x - x - 3$

$4x - 6 = 7x - 3$

$-7x -7x$

$-3x - 6 = -3$

$+6 \quad +6$

$\dfrac{-3x}{-3} = \dfrac{3}{-3} \quad x = -1$

Solution
$x = -1$

Linear 5. $0.5x - 0.3 = 1.7$

Clear decimal x10

$10 \cdot .5x - 10 \cdot .3 = 10 \cdot 1.7$

$5x - 3 = 17$

$+3 \quad +3$

$\dfrac{5x}{5} = \dfrac{20}{5}$

$x = 4$

Solution
$x = 4$

Linear 6. $0.05(x + 2) + 0.22 = 0.03(x + 6)$

Clear decimal x100

$100 \cdot .05(x + 2) + 100 \cdot .22 = 100 \cdot .03(x + 6)$

$5(x + 2) + 22 = 3(x + 6)$

$5x + 10 + 22 = 3x + 18$

$5x + 32 = 3x + 18$

$-3x -3x$

$2x + 32 = 18$

$-32 \quad -32$

$\dfrac{2x}{2} = \dfrac{-14}{2}$

$x = -7$

Solution
$x = -7$

Solve the rational or quadratic equations. For the rational equations write the value of the variable that makes the denominator zero.

Rational 7. $\frac{4}{x} + 2 = \frac{3}{2x}$

$x \neq 0$

LCM

$2x$

$2x \cdot \frac{4}{x} + 2x \cdot 2 = 2x \cdot \frac{3}{2x}$

$8 + 4x = 3$
$-8 \qquad -8$

$\frac{4x}{4} = \frac{-5}{4}$

$x = -\frac{5}{4}$

$\boxed{\text{Solution } x = -\frac{5}{4}}$

Rational 8. $\frac{7}{x-5} + 2 = \frac{x+2}{x-5}$

$x \neq 5$

LCM
$x-5$

$(x-5)\left(\frac{7}{x-5}\right) + 2(x-5) = (x-5)\left(\frac{x+2}{x-5}\right)$

$7 + 2x - 10 = x + 2$
$2x - 3 = x + 2$
$-x \qquad -x$
$x - 3 = 2$
$+3 \quad +3$
$x = 5$ → This value makes the denominator zero. Thus

$\boxed{\text{No Solution}}$

Rational 9. $\frac{2}{x+1} + \frac{3}{x^2-1} = \frac{5}{x-1}$

$x \neq 1$
$x \neq -1$

LCM
$(x+1)(x-1)$

$(x+1)(x-1) \cdot \frac{2}{x+1} + (x+1)(x-1) \cdot \frac{3}{(x+1)(x-1)} = (x+1)(x-1) \cdot \frac{5}{x-1}$

$2(x-1) + 3 = 5(x+1)$
$2x - 2 + 3 = 5x + 5$
$2x + 1 = 5x + 5$
$-2x \qquad -2x$
$1 = 3x + 5$
$-5 \qquad -5$
$-4 = 3x$

$\frac{-4}{3} = \frac{3x}{3}$

$x = \frac{-4}{3}$

$\boxed{\text{Solution } x = \frac{-4}{3}}$

Quadratic 10. $x(x+5) + x(x+1) = (2x+3)(x-4)$

$x^2 + 5x + x^2 + x = 2x^2 - 8x + 3x - 12$
$2x^2 + 6x = 2x^2 - 5x - 12$
$-2x^2 \qquad -2x^2$
$6x = -5x - 12$
$+5x \quad +5x$

$\frac{11x}{11} = \frac{-12}{11}$

$x = \frac{-12}{11}$

$\boxed{\text{Solution } x = \frac{-12}{11}}$

Solve the following equations. Indicate what the solutions is or if it is one of the special cases. (No Solution or All Real Numbers)

7. $3x + 2(x+4) = 5x - 7$

$3x + 2x + 8 = 5x - 7$
$5x + 8 = 5x - 7$
$-5x \qquad -5x$
$8 = -7$ ← variables cancelled
False statement
$\boxed{\text{No Solution}}$

8. $2x + 3x = 7x$

$5x = 7x$
$-7x \quad -7x$
$\frac{-2x}{-2} = \frac{0}{-2}$
$x = 0$

9. $5x - 3(x+1) = 2x - 3$

$5x - 3x - 3 = 2x - 3$
$2x - 3 = 2x - 3$
$-2x \qquad -2x$
$-3 = -3$ ← variables cancelled
True statement

$\boxed{\text{Solution All Real Numbers}}$