

A General Strategy for Factoring

1. Factor the Greatest Common Factor

2. Count the number of terms.

a. If there are **two terms**, determine if the problem is the special case involving two terms. The special cases are the following.

i. Difference of squares $a^2 - b^2 = (a + b)(a - b)$

ii. Sum of squares Not Factorable

iii. Difference of cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

iv. Sum of cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

b. If there are **three terms**, factor using the factoring trinomials by grouping method or the trial and error method.

c. If there are **four terms**, factor by grouping.

i. Group the first two together and the last two together.

ii. Group three of the terms together and leave one by itself.

Example: Factor the expression completely. $4x^3 - 10x^2$

$= 4x^3 - 10x^2$ Factor GCF first

$= 2x^2(2x - 5)$ The number of terms inside the parenthesis is 2.

$= 2x^2(2x - 5)$ Since $2x - 5$ is not a difference of squares the expression is factored completely

Example: Factor the expression completely. $30x^2 + 15x + 10xy + 5y$

$= 30x^2 + 15x + 10xy + 5y$ Factor GCF first.

$= 5(6x^2 + 3x + 2xy + y)$ The number of terms inside the parenthesis is 4.

Factor by grouping:

$= 5[(6x^2 + 3x) + (2xy + y)]$ 1) Group first 2 terms together and last 2 terms together.

$= 5[3x(2x + 1) + y(2x + 1)]$ 2) Factor the greatest common factor out of each grouping.

$= 5[(2x + 1)(3x + y)]$ 3) Factor the greatest common factor out of the remaining 2 terms.

$= 5(2x + 1)(3x + y)$

Example: Factor the expression completely. $12x^3 - 14x^2 - 6x$

$= 12x^3 - 14x^2 - 6x$ Factor the GCF first.

$= 2x(6x^2 - 7x - 3)$ The number of terms inside the parenthesis is 3.

Follow the method for factoring trinomials.

1) Multiply $6(-3) = -18$.

2) Find two numbers that multiply to -18 and add to -7.

Factors of -18

Sum of Factors of -18

$(1) \cdot (-18)$

$(1) + (-18) = -17$

$(-1) \cdot (18)$

$(-1) + (18) = 17$

$(2) \cdot (-9)$

$(2) + (-9) = -7$

$(-2) \cdot (9)$

$(-2) + (9) = 7$

$(3) \cdot (-6)$

$(3) + (-6) = -3$

$(-3) \cdot (6)$

$(-3) + (6) = 3$

$= 2x [6x^2 + 2x - 9x - 3]$

$= 2x [(6x^2 + 2x) + (-9x - 3)]$

$= 2x [2x(3x + 1) - 3(3x + 1)]$

$= 2x [(3x + 1)(2x - 3)]$

$= 2x(3x + 1)(2x - 3)$

Example: Factor the expression completely. $27xy^2 - 48x$

$= 27xy^2 - 48x$ Factor the GCF first.

$= 3x(9y^2 - 16)$ The number of terms inside the parenthesis is 2.

$= 3x(3y + 4)(3y - 4)$ Factor using difference of squares.